Wimborne Area Schools

Framework for Progression in Mathematical Calculation

St John's First School, Merley First School, , Pamphill First School, Wimborne First School, Witchampton First School, Allenbourn Middle School
With thanks to: Henbury View First School and Sturminster Marshall First School

Framework for Progression in Mathematical Calculation

Introduction

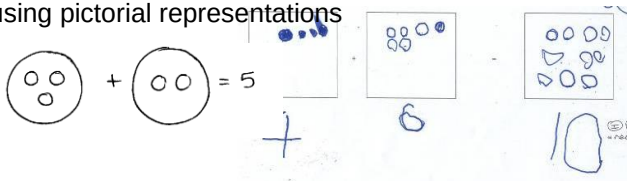
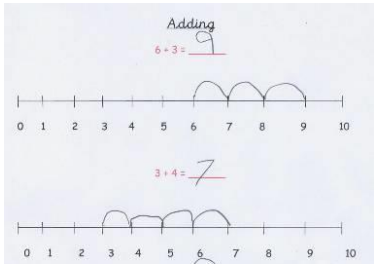
This document and the progressions for developing effective methods of calculation have been compiled by a consortium of first and middle schools from the Wimborne area and adopted as the agreed route to efficient calculation.

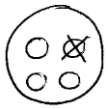
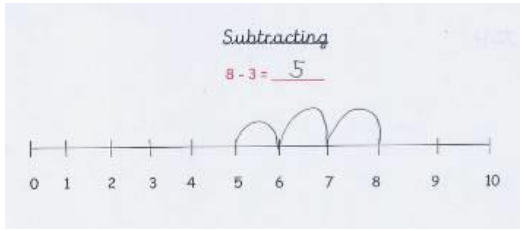
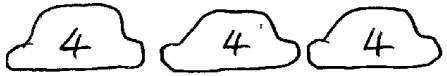
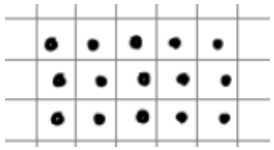
This framework details the key written methods of mathematical calculation to be taught. Its purpose is to promote a consistent and progressive approach to the teaching of mathematical calculation skills, in line with the expectations of the 2014 Mathematics curriculum. Although the main focus of this policy is on the progression to pencil and paper procedures it is important to recognise that the ability to calculate mentally underpins all calculation. Written calculation methods are not a replacement for mental calculation but structures to enable more complex calculations to be carried out efficiently. In every written method there is an element of mental processing. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

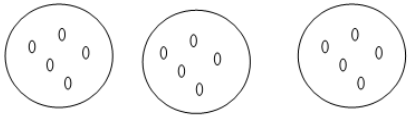
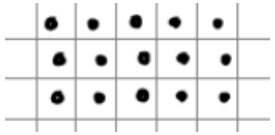
A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately. Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills, learned in working with a range of manipulative equipment, which provide the foundation for jottings and informal written methods of recording. These mental skills lead on to more formal written methods of calculation. Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When introducing a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept then, as competence increases, larger and more complex numbers can be tackled. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. The transition between years should not be hurried as not all children will be ready to move on to the next stage at the same time. Progression to the next stage should be made when mastery of the current stage is evident. Progression should be based on attainment across the mathematics curriculum and not just in the calculation processes. Judgements of mastery should be based on evidence gathered in routine classroom assessments and tasks of the pupils competence and accuracy in applying methods learned.

The long-term aim for our children is to have, and be able to select from, a variety of efficient (accurate, reliable and quick) methods of calculation that are appropriate to solve a range of calculation problems.
They should do this by always asking themselves:
• 'Can I do this in my head?'
• 'Can I do this in my head using drawings or jottings?'
• 'Do I need to use a written method?'

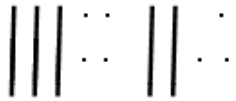
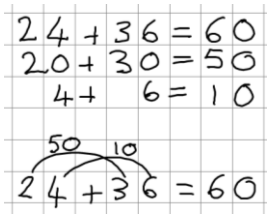
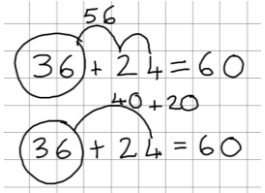
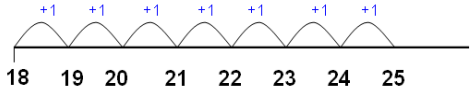
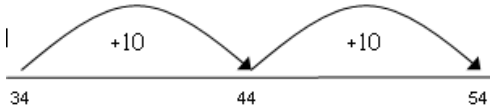
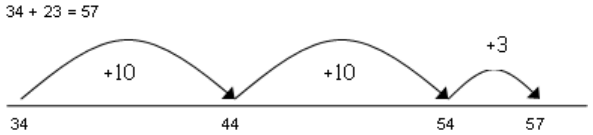
Calculation Framework (NC 2014)

	Addition Mentally (Including Jottings)	Addition Written
Year 1	• represent and use number bonds within 20 - represent number pairs to = 10 - use number pairs to = 10 to work out number pairs to = 20 - represent doubles up to double 10 - represent number bonds to make 3, 4, 5, 6, 7, 8, 9 in all ways using addition - use number bonds to make 3, 4, 5, 6, 7, 8, 9 • add one-digit and two-digit numbers to 20, including zero - using concrete objects (including exchanging Tens & Ones)  - using pictorial representations 	• read, write and interpret mathematical statements involving addition (+) and equals (=) signs 5 + 7 = • solve missing number problems (using numbers up to 20) 7 = __ + 2 

	Subtraction Mentally (Including Jottings)	Subtraction Written
	• represent and use subtraction facts within 20, relating them to the corresponding addition facts (see addition) - represent halves of even numbers up to 20 • subtract one-digit and two-digit numbers to 20, including zero - using concrete objects (including exchanging Tens & Ones)  - using pictorial representations  	• read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs 15 - 5 = • solve missing number problems (using numbers up to 20) 7 = ___ – 9 
Year 1	**Multiplication Mentally (Including Jottings)**	**Multiplication Written**
	• calculate the answer to multiplication problems (with the support of the teacher) (using numbers up to 20) - using concrete objects  - using pictorial representations  - using arrays 5 x 3 = 15 	(mental with jottings only)

	Division Mentally (Including Jottings)	Division Written
	• calculate the answer to division problems (with the support of the teacher) (using numbers up to 20) - using concrete objects  - using pictorial representations  - using arrays  15 ÷ 3 = 5	(mental with jottings only)

Calculation Framework (NC 2014)

	Addition Mentally (Including Jottings)	Addition Written
Year 2	• recall and use addition facts to 20 fluently, and derive and use related facts up to 100 - recall number pairs to = 10 - recall number pairs to = 20 - derive number pairs to = 100 in multiples of 10 - derive number pairs to = 100 e.g. 43 + ? = 100 - recall doubles up to double 10 - recall number bonds to make 3, 4, 5, 6, 7, 8, 9 in all ways using addition - derive additions of multiples of 10 • add a two-digit number and ones (using numbers up to 100) • add a two-digit number and tens (using numbers up to 100) • add two two-digit numbers (using numbers up to 100) • add three one-digit numbers - using concrete objects (including exchanging Tens & Ones)  - using pictorial representations  - mentally (with jottings)  	• add a two-digit number and ones (using numbers up to 100) (no bridging the tens, bridging the tens) 18+7=  18 + 7 25 Calculation to be set out in columns to support the understanding of place value. However, this should be solved using a range of strategies, not standard column method. e.g. Number line, Dienes, Numicon, diagrams and jottings. Partitioning & exchanging should be included. • add a two-digit number and tens (using numbers up to 100) 34 + 20 =  34 + 20 NOTE: See text box above. • add two two-digit numbers (using numbers up to 100) (no bridging the tens, bridging the tens)  34 + 23 67 + 29 NOTE: See text box above.

Year 2	Subtraction Mentally (Including Jottings)	Subtraction Written

Subtraction Mentally (Including Jottings)

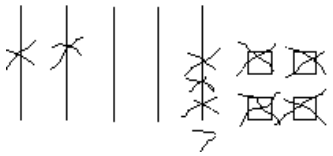
- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100

- use number bonds to make 3, 4, 5, 6, 7, 8, 9 to work out subtractions i.e. use reversals.
- represent 5 + 3 = 8, therefore 8 – 3 = 5
- recall halves of even numbers up to 20

- subtract a two-digit number and ones
- subtract a two-digit number and tens
- subtract two two-digit numbers
- subtract three one-digit numbers

- using concrete objects (including exchanging Tens & Units)

- using pictorial representations

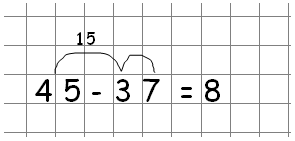


- mentally (with jottings)

54 – 27 54 – 20 = 34

- 4 = 30

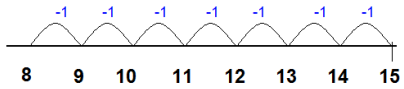
- 3 = 27



Subtraction Written

- subtract a two-digit number and ones (no bridging the tens, bridging the tens)

15-7=

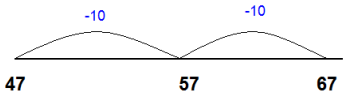


$$\begin{array}{r} 15 \\ -\ 7 \\ \hline \end{array}$$

Calculation to be set out in columns to support the understanding of place value.
However, this should be solved using a range of strategies, not standard column method.
e.g. Number line, Dienes, Numicon, diagrams and jottings.
Partitioning & exchanging should be included.

- subtract a two-digit number and tens

67 – 20 =

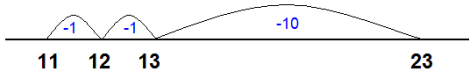


$$\begin{array}{r} 67 \\ -\ 20 \\ \hline \end{array}$$

NOTE: See text box above.

- subtract two two-digit numbers (no bridging the tens, bridging the tens)

23 – 12 =



$$\begin{array}{r} 23 \\ -\ 12 \\ \hline \end{array} \qquad \begin{array}{r} 43 \\ -\ 27 \\ \hline \end{array}$$

NOTE: See text box above.

Year 2

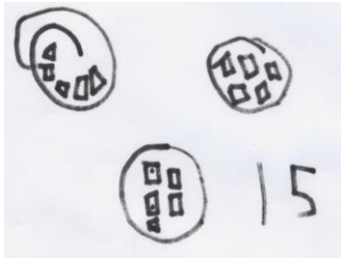
Multiplication Mentally (Including Jottings)

- recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- recognise odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables

- using materials

- using mental methods (with jottings)

5, 10, 15, 20, 25



Multiplication Written

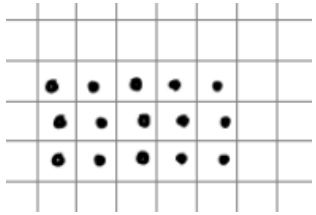
- write calculations using the multiplication (×) and equals (=) signs

4 x 5 = 20

- calculate mathematical statements for multiplication within the multiplication tables

- using arrays

3 x 5 = 15



- using repeated addition:

4 x 5 =

5 + 5 + 5 + 5 = 20

This can be done on a number line.

Division Mentally (Including Jottings)

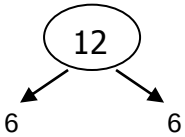
- recall and use division facts for the 2, 5 and 10 multiplication tables

- use 2x table to divide by 2, counting in multiples or reversing the calculation
- use 10x table to divide by 10, counting in multiples or reversing the calculation
- use 5x table to divide by 5, counting in multiples or reversing the calculation

- calculate mathematical statements for division within the multiplication tables

- using materials

- using mental methods (with jottings)



Division Written

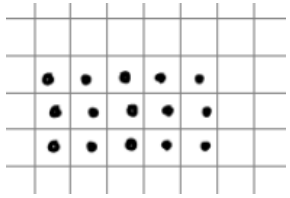
- write calculations using the division (÷) and equals (=) signs

30 ÷ 2 = 15

- calculate mathematical statements for division within the multiplication tables (i.e. no remainders)

- using arrays

15 ÷ 5 = 3



- using repeated addition

15 ÷ 5 =
5 10 15

This can be done on a number line.

Calculation Framework (NC 2014)

<table>
<tr><th></th><th>Addition Mentally (Including Jottings)</th><th>Addition Written</th></tr>
<tr><td rowspan="2">Year 3</td><td>

add a three-digit number and ones (using numbers up to 1000)
add a three-digit number and tens (using numbers up to 1000)
add a three-digit number and hundreds (using numbers up to 1000)

(no bridging the hundreds, bridging the hundreds)
</td><td>

add numbers with up to three digits, using formal written methods of columnar addition

(No exchanging, exchanging ones to tens, exchanging tens to hundreds)
45 + 22 = 67
48 + 3 = 51 (1)
48 + 93 = 141 (1 1)
748 + 93 = 841 (1 1)
658 + 169 = 827 (1 1)
Apparatus should be available to support children's standard method calculations
</td></tr>
<tr><th>Subtraction Mentally (Including Jottings)</th><th>Subtraction Written</th></tr>
<tr><td>

subtract a three-digit number and ones
subtract a three-digit number and tens
subtract a three-digit number and hundreds

(no bridging the hundreds, bridging the hundreds)
</td><td>

subtract numbers with up to three digits, using formal written methods of columnar subtraction

(No exchanging, exchanging tens to ones, exchanging hundreds to tens)
48 - 27 = 21
48 - 9 = 39
48 - 29 = 19
348 - 29 = 319
348 - 269 = 79
Apparatus should be available to support children's standard method calculations
</td></tr>
</table>

Multiplication Mentally (Including Jottings)

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental strategies

35 x 4

70 x 2 = 140

120
35 x 4
20

35 x 4
3 x 4 = 12
5 x 4 = 20
140

12 x 10 = 120

Multiplication Written

- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers progressing to formal written methods

(No exchanging, exchanging ones to tens, exchanging tens to hundreds)

```
  1 3      2 8      2 4
x   3    x   3    x   8
  3 9      8 4    1 9 2
           2        3
```

Children should be secure on mental methods of multiplication with jottings before moving on to this standard written method

Division Mentally (Including Jottings)

- recall and use division facts for the 3, 4 and 8 multiplication tables

27 ÷ 3 3, 6, 9, 12, 15, 18, 21, 24, 27 9 x 3 = 27

Division Written

- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers (no remainders, remainders)

```
64 ÷ 4 = 16
- 40      10 x 4
  24
- 24       6 x 4
```

```
64 ÷ 4 = 16
- 40      10 x 4
  24
- 24       6 x 4
```

Children should be secure on mental methods of division with jottings before moving on to this chunking method

Calculation Framework (NC 2014)

Year 4

Addition Mentally (Including Jottings)

It would be helpful if children could begin to add 4 digit numbers mentally where appropriate, as Yr 3 add 3 digit and Yr 5 add 5 digit. (no bridging, bridging) (include jottings where necessary)
4 digit + 1s
4 digit + 10s
4 digit + 100s

Addition Written

- add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate

(Exchanging ones to tens, exchanging tens to hundreds, exchanging hundreds to thousands)

```
  2345        2355        2855
+   46      +  456      + 3766
  2391        2811        6621
    1         1 1         1 1 1
```

Apparatus should be available to support children's standard method calculations

Subtraction Mentally (Including Jottings)

It would be helpful if children could begin to subtract 4 digit numbers mentally where appropriate, as Yr 3 subtract 3 digit and Yr 5 subtract 5 digit. (no bridging, bridging) (include jottings where necessary)
4 digit - 1s
4 digit - 10s
4 digit - 100s
Finding the difference between 2 larger numbers close together can be taught by counting/jumping up from the lowest. 1000 – 998 = 2

Subtraction Written

- subtract numbers with up to 4 digits using the formal written methods of columnar and subtraction where appropriate

(Exchanging tens to ones, exchanging hundreds to tens, exchanging thousands to hundreds)

```
    3 1          2 1          1 1
  2 3 4 5      2 3 4 5      2 3 4 5
- 1 2 7      - 1 6 3      - 1 5 2 3
  2 2 1 8      2 1 8 2      0 8 2 2
```

Multiplication Mentally (Including Jottings)

- recall multiplication facts for multiplication tables up to 12 × 12 (6,7,9,11,12)
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers

180
35 x 6
30

35 x 6
3 x 6 = 18
5 x 6 = 30
210

18 x 10 = 180

Multiplication Written

- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

```
    2 4
×     6
-------
  1 4 4
-------
    2
```

	Division Mentally (Includeing Jottings)	Division Written
	• recall division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to divide mentally 240 ÷ 6 = 24 ÷ 6 = 4 240 ÷ 6 = 40 • divide by 1 multiply together three numbers	• write and calculate mathematical statements for division using the multiplication tables that they know including a 3 digit number divided by a 2 digit number. (no remainders, remainders) 846 ÷ 6 = - 600 100 x 6 - 60 10 x 6 - 60 10 x 6 - 60 10 x 6 - 60 10 x 6 - 6 1 x 6 Answer to each subtraction can be added in to the calculation if required. See Yr3.

Calculation Framework (NC 2014)

<table>
<tr><th></th><th>Addition Mentally (Including Jottings)</th><th>Addition Written</th></tr>
<tr><td rowspan="2">Year 5</td><td>• add numbers mentally with increasingly large numbers

10573 + 3200 = 13773

• add numbers to 2 decimal places

9.58 + 3.08 = 12.66</td><td>• add whole numbers with more than 4 digits, including using efficient written methods (columnar addition)
44388
+ 5896
50284
1 1 1 1
• add numbers to 3 decimal places
32.148
+ 9.738
41.886
1 1</td></tr>
<tr><th>Subtraction Mentally (Including Jottings)</th><th>Subtraction Written</th></tr>
<tr><td>• subtract numbers mentally with increasingly large numbers

64501 – 4300 = 60201

• subtract numbers to 2 decimal place

7.47 – 3.15 = 4.32</td><td>• subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction)
5 13 1
6 4 6 7
- 2 6 8 4
3 7 8 3
• subtract numbers to 3 decimal places
2 1 1
32.148
- 9.738
22.410</td></tr>
</table>

Multiplication Mentally (Including Jottings)

- multiply and divide numbers mentally drawing upon known facts

45 x 6 =
45 x 2 = 90, 90 x 3 (9x3x10)
= 270
38 x 15 =
38 x 10= 380 + 190 (1/2 of 380)
= 570

- solve problems involving multiplication where larger numbers are used by decomposing them into their factors

350 x 25 = 7 x 5 x 10 x 5 x 5

- multiply whole numbers and those involving decimals by 10, 100 and 1000

234.7 x 10 becomes
2347.0
9067.43 x 100 becomes
906743.0

When multiplying by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the left for each multiple of 10

Multiplication Written

- multiply numbers up to 4 digits by a one-digit number using an efficient written method

```
  4346
   x  8
 34768
 2 3 4
```

- multiply numbers up to 4 digits by a two-digit number using long multiplication 4115 x 25

```
  4115
   x 25
   1
 82300   (x10, x2)
    2
 20575
102875
```

recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})

2^2 = 2 x 2 = 4
3^3 = 3 x 3 x 3 = 27

Division Mentally (Including Jottings)

- divide numbers mentally drawing upon known facts
- multiply and divide whole numbers involving decimals by 10, 100 and 1000

234.7 ÷ 10 becomes
23.47

9067.43 ÷ 100
becomes
90.6743

When dividing by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the right for each multiple of 10

Division Written

- divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context

72 ÷ 5

```
      1 4 r 2
   ---------
5 ) 7 2
    2 2
```

1368 ÷ 9

```
      152
   ---------
9 ) 1368
    4 1
```

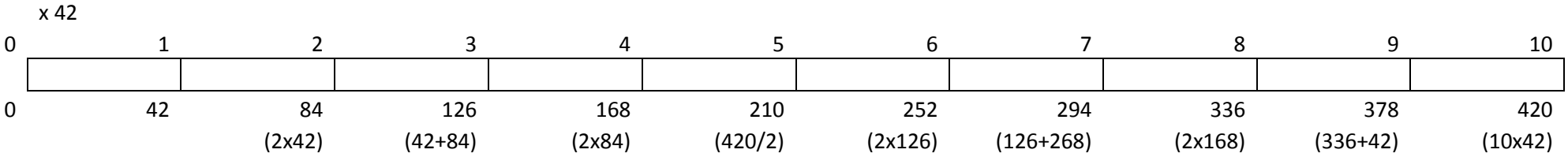
Calculation Framework (NC 2014)

	Addition Mentally (Including Jottings)	Addition Written
Year 6	• add numbers mentally with increasingly large numbers	• add whole numbers with more than 4 digits, including using efficient written methods (columnar addition) (See Year 5 for examples)
	Subtraction Mentally (Including Jottings)	**Subtraction Written**
	• subtract numbers mentally with increasingly large numbers	• subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction) (See Year 5 for examples)
	Multiplication Mentally (Including Jottings)	**Multiplication Written**
	• perform mental calculations, including with mixed operations and large numbers	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication

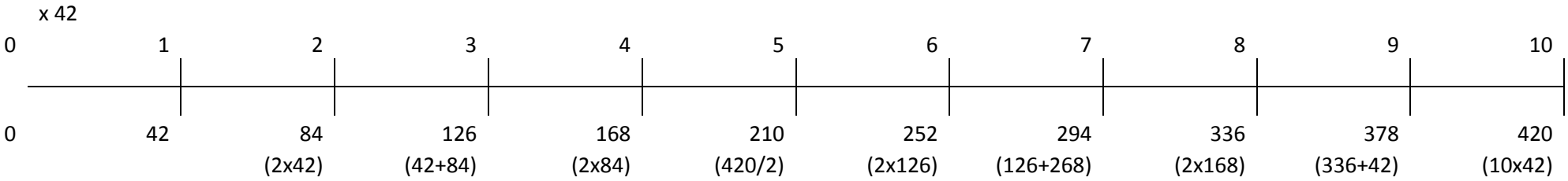
	Division Mentally (Including Jottings)	Division Written
	• perform mental calculations, including with mixed operations and large numbers	• divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context 2666 ÷ 42 Remainder 63 r20 42)2666 252 146 126 20 *10 x 42= 420* *2 x42 = 84* *4 x42 = 168* *5 x 42 = 210* *6 x 42 = 252* Fraction 63 20/42 42)2666 252 146 126 20 Use doubling/halving to find other multiples. **See Appendix 1** for Bar and Double Number Line representations Decimal 63.47 42)2666*00* -252 146 -126 200 -168 320 -294 26

Tools to aid multiplication in division calculations

Bar Method



Double Number Line Method



This method of multiplication uses know or easily computable facts, by doubling and halving or addition, to construct the data needed to carry out the calculation.
There is no need to calculate all values from 1 times to 10 times, only those needed.